JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 19, No. 2, March–April 1996

Adaptation of Centers of Approximation for Nonlinear Tracking Control

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Sufficient conditions for the asymptotic stability and exponential convergence of tracking control for nonlinear systems have proliferated in recent research. Various theories for stability have been related to, and rely upon, the functional form of the governing nonlinear equations, the linearizability of the governing equations by canonical (Lie) transformations, the linear appearance of adaptive parameters, and the richness of the input excitation. Previously, sufficient conditions have been derived for a class of Hamiltonian adaptive tracking control problems using radial basis function approximants. We demonstrate that a crucial element determining the performance of this class of tracking control methodologies is the location of the centers of approximation in the radial basis approximant/network. A learning vector quantization algorithm is employed to simultaneously adapt both the centers of approximation and the feedback gains. The simultaneous adaptation of both approximant centers in configuration space and feedback gains can demonstrate remarkable improvements in convergence rate, as compared to adaptation of feedback gains alone.

Introduction

THE evolutionary nature of large space structures results in control design tasks of unprecedented difficulty. These systems will require in-orbit construction, are highly flexible in nature, use onboard, large articulated manipulator systems, must meet fine line-of-sight pointing requirements, and must be fault tolerant with respect to structural and control system damage. As a result, the governing system equations are time varying and can be highly nonlinear. Furthermore, robust control strategies alone cannot accommodate the anticipated variations in system configuration. For these reasons, adaptive control will play a dominant role in the control of these integrated, evolutionary structures. As an example, consider the phase 2 configuration of the NASA Langley Control Structure Interaction (CSI) evolutionary test article depicted in Fig. 1.

In this configuration, two pointing devices and one scanning device mounted on two-axis gimbals are attached to the primary structure. As the primary structure undergoes structural deformation, the pointing sensors must meet stringent line-of-sight performance criteria, whereas the third sensor must perform a raster-scan maneuver. Each of the three scanning devices can be represented by low-dimensional, nonlinear equations of motion, whereas the overall system has thousands of degrees of freedom.

Although significant progress has been made for adaptive control of classes of nonlinear systems,[1–5] this paper extends the work in Kurdila[6] to obtain adaptive Hamiltonian control strategies employing reduced order estimation techniques. The reduction in order is achieved using radial basis function approximants and simultaneously adapting both feedback gains and centers of approximation in configuration space. Preliminary studies indicate that the method can achieve remarkable acceleration in the rate of convergence to the desired trajectory, as compared to radial basis networks with fixed centers of approximation. The technique also serves as a means of addressing the high dimensionality noted by Narendra and Parthasarathy[1] associated with multidimensional radial basis function approximants.

Adaptive Hamiltonian Tracking Control

In Kurdila,[6] adaptive control strategies are derived for the class of Hamiltonian systems introduced by Nijmeijer and Van Der Schaft[7] and Van Der Schaft[8]:

$$\dot{q}_i = \frac{\partial H_0}{\partial p_i} - \sum_1^m \frac{\partial H_j}{\partial p_i} u_j \tag{1a}$$

$$\dot{p}_i = -\frac{\partial H_0}{\partial q_i} + \sum_1^m \frac{\partial H_j}{\partial p_i} u_j \tag{1b}$$

In these equations, H_0 represents the internal, or open-loop, system Hamiltonian. The functions $\{u_1(t), \ldots, u_m(t)\}$ represent the input controls, whereas the functions

$$Y_j = H_j(q, p) \qquad j = 1, m \tag{2}$$

are the natural outputs associated with preceding Hamiltonian system. The observations $Y_j(t)$ are denoted natural in that a simple work-energy principle can be derived for the closed-loop system just described[8]:

$$\frac{\mathrm{d}H_0}{\mathrm{d}t} = \sum_1^m \frac{\mathrm{d}H_k}{\mathrm{d}t} u_k \tag{3}$$

One advantage of this representation system dynamics is that integrals of motion and potential Lyapunov functions for the closed-loop system are easily constructed from the given identity. Also note that the system constitutes a generalization of symmetric output feedback control for linear systems.[9]

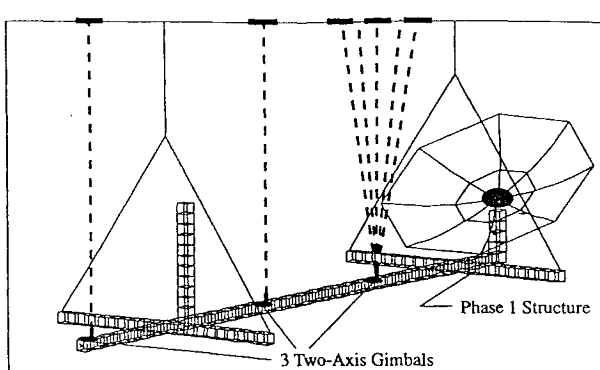

Fig. 1 NASA Langley CSI evolutionary test structure.

Received April 19, 1993; revision received July 29, 1995; accepted for publication Aug. 30, 1995.

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The primary theoretical result in Kurdila[6] is embodied in the following theorem:

Theorem.[6] Consider the Hamiltonian control system described in Eqs. (1) and its associated outputs Y_j in Eq. (2). Suppose that the feedback control has been selected as

$$u_j(t) = -(1/\varepsilon)(\alpha_j H_j + \beta_j \dot{H}_j + \gamma_j \ddot{H}_j) \tag{4}$$

Then we have the following three theorem parts.

a) The tracking error lies within a hyperellipsoid in phase space whose diameter is $0(I/\varepsilon)$:

$$\sum \frac{H_j^2}{1/\alpha_j} + \sum \frac{\dot{H}_j^2}{1/\gamma_j} \le 2\varepsilon H_0(0) \tag{5a}$$

b) If $\beta \neq 0$, then the tracking error asymptotically converges to zero in phase space:

$$(||H_j|| \to 0) \text{ and } (||\dot{H}_j|| \to 0) \tag{5b}$$

c) If $\beta = 0$ and $H_0(t) \to H_0(\infty) \equiv$ const, then the tracking error asymptotically approaches the surface of the ellipsoid:

$$\sum \frac{H_j^2}{1/\alpha_j} + \sum \frac{\dot{H}_j^2}{1/\gamma_j} = 2\varepsilon[H_0(0) - H_0(\infty)] \tag{5c}$$

The proof of theorem is as follows. Substituting the control of Eq. (4) into the identity for natural Hamiltonian systems in Eq. (3),

$$\frac{\mathrm{d}}{\mathrm{d}t} H_0 + \sum_j \dot{H}_j \frac{(\alpha_j H_j + \beta_j \dot{H}_j + \gamma_j \ddot{H}_j)}{\varepsilon} = 0 \tag{6}$$

Integrating Eq. (6),

$$2\varepsilon H_0(t) + \sum_j \alpha_j H_j^2 + \sum_j \gamma_j \dot{H}_j^2 = 2\varepsilon H_0(0) - 2\sum_j \int_0^t \beta_j \dot{H}_j^2 \mathrm{d}\tau = 0 \tag{7}$$

From Eq. (7), theorem parts (5a–5c) can easily be seen. It follows straightforward that stability is assured ($\dot{V} < 0$) when a Lyapunov function of the form

$$V = H_0 + \sum_j \frac{\alpha_j H_j^2}{2\varepsilon} + \sum_j \frac{\gamma_j \dot{H}_j^2}{2\varepsilon} \tag{8}$$

is chosen.

Now for the tracking control problem, define the control u_j as

$$u_j = \sum_k c_{jk}(t) W_{jk}(q, p) \tag{9}$$

and assume that the system output can be approximated by a radial basis function network as

$$-\frac{\gamma \ddot{H}_j + \beta \dot{H}_j + \alpha H_j}{\varepsilon} \sum_k c_{jk}(t) W_{jk}(q, p) + \Delta_j \tag{10}$$

where $c_{jk}(t)$ are time-varying adaptive feedback gains, $W_{jk}[q(t), p(t)]$ are shape functions in phase space, and Δ_j is the error in approximation. Substituting Eqs. (9) and (10) into the identity Eq. (3),

$$\frac{\mathrm{d}}{\mathrm{d}t} H_0 = -\sum_j \left(\frac{\gamma \ddot{H}_j + \beta \dot{H}_j + \alpha H_j}{\varepsilon} \right) \dot{H}_j - \sum_j \dot{H}_j \Delta_j \tag{11}$$

Integrating Eq. (11),

$$H_0(t) + \sum_j \frac{\alpha H_j^2}{2\varepsilon} + \sum_j \frac{\gamma \dot{H}_j^2}{2\varepsilon} = H_0(0) - \sum_j \int_0^t \frac{\beta \dot{H}_j^2}{\varepsilon} \mathrm{d}\tau - \sum_j \int_0^t \dot{H}_j \Delta_j \,\mathrm{d}\tau \tag{12}$$

Defining

$$\psi_j(t) = \frac{(\gamma_j \ddot{H}_j + \beta_j \dot{H}_j + \alpha_j H_j)}{\varepsilon} = \sum_k c_{jk}(t) W_{jk}(q, p) + \Delta_j \tag{13}$$

and supposing that γ, β, and α are selected so that the transfer function $\Gamma(s)$ defined from

$$H_j(s) = \Gamma(s)\psi_j(s) \tag{14}$$

is strictly positive real, $\Gamma(s)$ has a state-space realization (A, B, C) by the Popov–Kalman–Yakubovitch lemma as

$$\dot{e}_j = A e_j + B\psi_j \tag{15}$$

$$H_j = C_{ej} \tag{16}$$

where $e_j, e_j, B, C^T \in \mathcal{R}^{m\times 1}$ and $A \in \mathcal{R}^{m\times m}$ with

$$PA + A^T P = -Q \tag{17}$$

$$PB = C^T \tag{18}$$

for stability. In the preceding equations, e_j represents an internal error.

Consider the candidate Lyapunov function

$$V = \sum_j \left\{ \left(\frac{\gamma \dot{H}_j^2}{2} + \frac{\alpha H_j^2}{2} \right) + \frac{\eta e_j^T P e_j}{2} + \sum_k \frac{c_{jk}^2}{2} \right\} \tag{19}$$

$$V = \varepsilon[H_0(0) - H_0(t)] - \sum_j \int_0^t \beta H_j^2 \mathrm{d}\tau - \varepsilon \sum_j \int_0^t \dot{H}_j \Delta_j \,\mathrm{d}\tau + \sum_j \frac{\eta e_j^T P e_j}{2} \sum_j \sum_k \frac{c_{jk}^2}{2} \tag{20}$$

Taking the derivative of the fifth term of the Lyapunov function in Eq. (20) while using the relations of Eqs. (17) and (18) leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(e_j^T P e_j\right) = -e_j^T Q e_j + 2 e_j^T C^T \psi_j \tag{21}$$

The derivative of the last term in the Lyapunov expression [Eq. (20)] is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_j \sum_k \frac{c_{jk}^2}{2} \right) = \sum_j \sum_k \dot{c}_{jk} c_{jk} \tag{22}$$

By choosing a gradient descent update law of the form

$$\dot{c}_{jk} = -\eta H_j W_{jk} \tag{23}$$

and making use of Eqs. (13) and (16), the derivative becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_j \sum_k \frac{c_{jk}^2}{2} \right) = -\eta \left(\sum_j \psi_j C e_j + \sum_j \Delta_j H_j \right) \tag{24}$$

This leads to the final form of $\dot{V}$, namely,

$$\dot{V} = -\sum_j \beta H_j^2 - \eta \sum_j \frac{e_j^T Q e_j}{2} + \mathcal{O}(\varepsilon, \Delta) \tag{25}$$

where the first two terms are negative and the third term is small. Therefore, $\dot{V} < 0$, and the system is stable.

Radial Basis Function Approximants

The statement of stability and convergence in the theorem does not specify a functional form for the shape functions in phase space. From the statement of the stability theorem, however, the approximation properties of these functions are crucial in establishing that the tracking control is asymptotically stable. Specifically, it is required that the approximation error

$$\psi_j(t) - u_j = \Delta_j$$

be small enough. In Narendra and Parthasarathy,[1] Kurdila,[6] and Poggio and Girosi,[10–12] these basis functions have been selected to be radial basis functions. There is considerable empirical evidence that these functions provide sufficient accuracy, although little rigorous investigation of the issue of accuracy has been carried out in the control literature.

On the other hand, significant progress has been made in characterizing the approximation properties of radial basis functions.[13] For example, the best approximant (in an L_2 sense) of a function $y = F(x)$ can be expressed as

$$y = F(x) = \sum_{\alpha} c_\alpha W(||x - \xi_\alpha||) \tag{26}$$

where $W(\cdot)$ can be selected from any of several candidate radial functions

$$W(r) = e^{-(r/c)^2} \qquad W(r) = (c^2 + r^2)^{-k}$$

$$W(r) = (c^2 + r^2)^j$$

Imposing the (numerous) data pairs collected from the experiment

$$y_i = F(x_i) \tag{27}$$

leads to the overdetermined system of equations

$$y_i = F(x_i) = \sum_{\alpha} c_\alpha W(||x_i - \xi_\alpha||) \tag{28}$$

In matrix form, these equations can be written

$$y = [W]c \tag{29}$$

where W is a $S \times N$ matrix with $S \gg N$. The rectangular matrix

$$[W]_{i\alpha} = W(||x_i - \xi_\alpha||) \tag{30}$$

cannot be inverted in classical sense, although the preceding system of equations can be solved via the pseudoinverse. (Naturally, the actual calculation would be carried out using a numerically stable singular value decomposition or QR algorithm.) Thus,

$$[W]^\dagger = ([W]^T[W])^{-1}[W]^T \tag{31}$$

$$c = [W]^\dagger y \tag{32}$$

The numerical stability of this approximation problem has been discussed in detail in Narcowich and Ward.[14] They have shown that the norm of the pseudoinverse $[W]$ is bounded above by a constant that depends on the radial basis function employed, the minimal separation of the data, and the dimension of the space in which the data lie:

$$||[W]^\dagger|| \leq c[W(x), \delta_\xi, N] \tag{33}$$

In a later section, it will be suggested how these results can be extended in principle to the tracking problem.

Perhaps one of the greatest motivations for using radial basis function approximants in the context of adaptive of tracking control, as in Refs. 1, 2, 6, and 10–12, is that they have two distinct, and useful characterizations. These two realizations are denoted the geometric and the network representations and are depicted in Figs. 2 and 3, respectively. The geometric realization has the advantage that it enables a clear visualization of the selected functions. Each center locates a single, bell-shaped function that effectively provides a localized representation of feedback gain in a region of configuration space. The network realization, on the other hand, illustrates that the feedback gains can be interpreted as the connection weights in a single-layer neural network.[10–13] This has important implications in hardware implementations of the adaptive control algorithms.

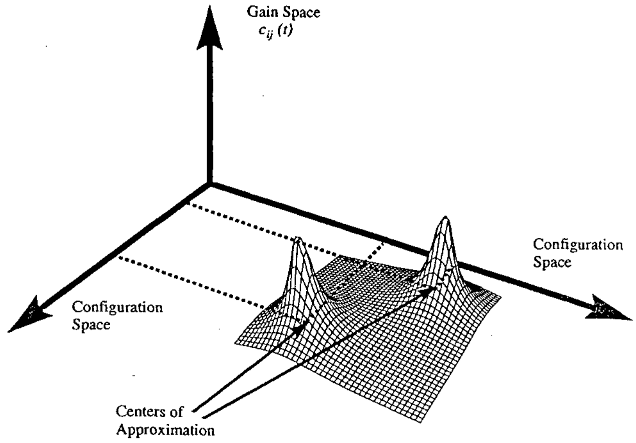

Fig. 2 Geometric realization of radial basis functions.

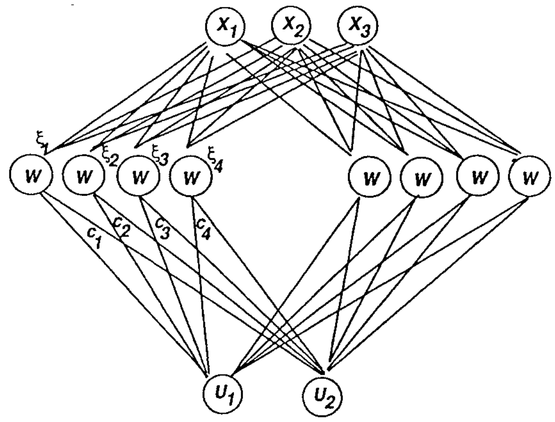

Fig. 3 Network realization of radial basis functions.

Center Adaptation by Vector Quantization

Although excellent results can be achieved in adaptive tracking control using radial basis function approximants, there are at least two related issues that must be resolved before the radial basis approximant method can be implemented in pragmatic control problems. Both of these issues are discussed in more detail elsewhere.[1,10–12]

1) Location of centers: As shown in Kurdila[6] and illustrated later in this paper, the fidelity of approximation absolutely requires that one places the centers of approximation judiciously. Moreover, the placement of centers that are fixed throughout the adaptation process requires a priori knowledge of the location of the system trajectories in phase (or configuration) space. Simply put, one must grid up those portions of phase space that will be visited frequently by the system trajectories. As opposed to, say, elliptic partial differential equations, this domain of definition is not, in general, known a priori.

2) Dimensionality of approximant: To obtain the precise approximations desired, Narendra and Parthasarathy[1] have noted that the dimensionality of the approximation can be prohibitively expensive for large problems. In fact, they advocate alternative functional approximation methods for problems of moderate dimensionality. Roughly speaking, if the dimension of the measurement space is m, and N_f functions are distributed uniformly in each dimension, the approximant involves on the order of $(N_f)^m$ radial basis function regressors.

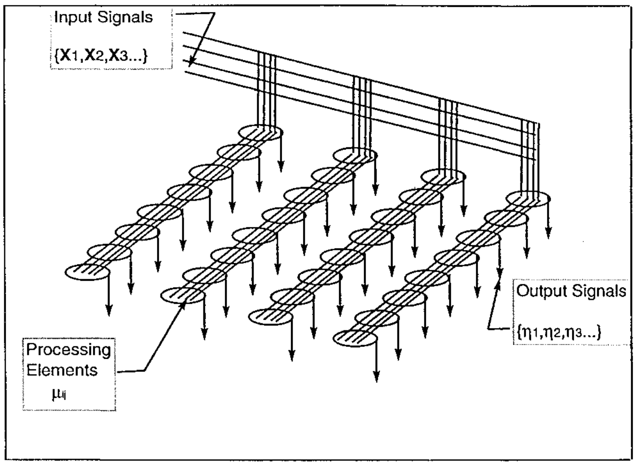

Fig. 4 Kohonen neural network schematic diagram.

However, Poggio and Girosi[10–12] suggest an alternative architecture for radial basis function approximation that has the potential for alleviating these two difficulties. That is, one can choose to adapt not only the feedback gains, but also the centers of approximation themselves. In this way, fewer functions should suffice to locally approximate a function only in the portions of the phase space domain that are visited by the system trajectories. The approach taken in this development is to utilize vector quantization error minimization based upon Kohonen's[15] topology preserving mappings.

A detailed presentation of Kohonen's[15] topology preserving mapping and its ability to self-organize would require a great deal of rather technical discussion. Perhaps the simplest representation of Kohonen's network is given in pp. 130–132 of Ref. 16. To simplify the discussion, only adaptation of the centers in configuration space, as opposed to phase space, is considered in the remainder of this section. In this presentation, one can visualize the neural network as receiving a set of scalar inputs

$$q(t) = \{q_1(t), \ldots, q_N(t)\}^T \in \mathcal{R}^N$$

and distributing these signals to an array of processing units

$$[\mu_{ij}] \in \mathcal{R}^{N \times N}$$

The output of the processing units is simply

$$\eta_i = \sum_j \mu_{ij} q_j \tag{34}$$

in this crude model, as depicted in Fig. 4. With this notation, the goal of Kohonen's method can be stated rather succinctly: "... to devise adaptive processes in which the parameters of all the (processing) units converge to such values that every unit becomes specifically matched ... to a particular domain of input signals ...."[15]

This statement can be clarified by the observation that if one denotes the rows

$$[\mu_{ij}] = \begin{bmatrix} \xi_1^T \\ \xi_N^T \end{bmatrix} \qquad \xi_i^T = [\mu_{i1} \ldots \mu_{iN}]$$

then

$$\eta_i = \xi_i^T q \tag{35}$$

is a measure of the similarity between signal and row ξ_i^T; it is just the inner product of vectors. Of course, this analogy can be extended only so far. The processor μ_{ij} adapt as they are presented different input signals and, therefore, can be viewed as functions of (discrete) time. In addition, the evolution, adaptation, laws for learning are complicated, vary among authors, and difficult to prove their convergence.

Fortunately, the algorithm describing the adaptation laws is quite simple. An example from Kohonen[15] can be summarized as follows.

1) Choose a size of a neighborhood of processors N_c.

2) Find the best similarity match for the input signal

$$||q(t_k) - \xi_c(t_k)|| = (\min/i < N)||q(t_k) - \xi_i(t_k)||$$

3) Update all processor units in the neighborhood of the best match

$$\xi_i(t_{k+1}) = \xi_i t_k + \alpha(t_k)\{q(t_k) - \xi_i(t_k)\}$$

4) Repeat for all input signals.

While the discussions thus far have been quite general, there is a probabilistic interpretation of the weight vectors ξ_i that should be noted. If one defines the average quantization error

$$E = \int_{V_q} ||q - \xi_{c(x)}||^r P(q)\, \mathrm{d}V_q \tag{36}$$

where the input vector is $q \in \mathcal{R}^N$, the volume differential is $\mathrm{d}V_q$, the index of closest weight vector to q is $c(q)$, and the point density of weight vectors is $P(q)$.

Then it can be shown from the algorithm that "... the asymptotic values of the ξ_i ... minimize E" when $N_c = \{c\}$ (Ref. 15). In other words, the ξ_k cluster to minimize the probability of quantization error.[15] The incorporation of the quantization error minimization algorithm into the adaptive Hamiltonian control strategy is straightforward. In this case, the closed-loop equations (1), (9), and (23) are considered as parameterized by the centers of adaptation ξ_{jk} as

$$\dot{q}_i = \frac{\partial H_0}{\partial p_i} - \sum_1^m \frac{\partial H_j}{\partial p_i} u_j \tag{37a}$$

$$\dot{p}_i = -\frac{\partial H_0}{\partial q_i} + \sum_1^m \frac{\partial H_j}{\partial q_i} u_j \tag{37b}$$

$$u_j(t) = \sum_k c_{jk}(t) W_{jk}[q(t), p(t); \xi_{jk}] \tag{38}$$

$$\dot{c}_{jk} = -\eta H_j(t) W_{jk}[q(t), p(t); \xi_{jk}] \tag{39}$$

The center evolution law now bears a close resemblance to simulated annealing algorithms wherein

$$\dot{\xi}_{jk}(t) = \begin{cases} \alpha(t)(q(t) - \xi_{jk}(t)) & jk \in N_{c(q)} \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

Numerical Example

A simple example has been selected to illustrate the profound effect the centers of approximation can have on the rate of convergence of the tracking control. The example also depicts the acceleration of

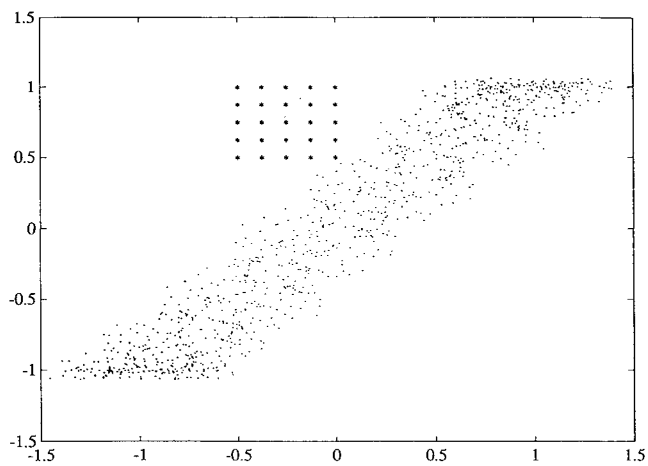

Fig. 5 Trajectory history and constant center grid (case 1).

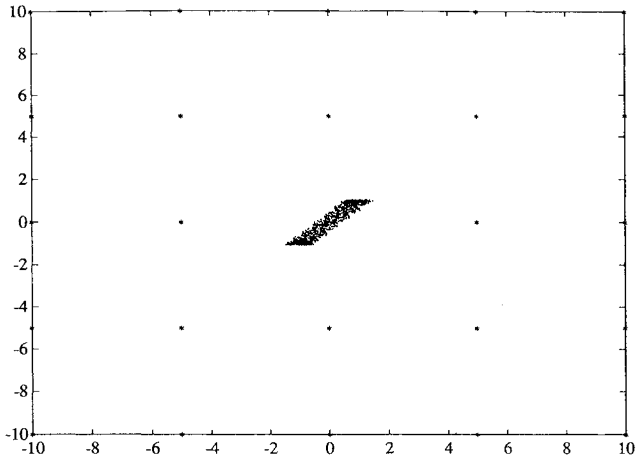

Fig. 6 Trajectory history and constant center grid (case 2).

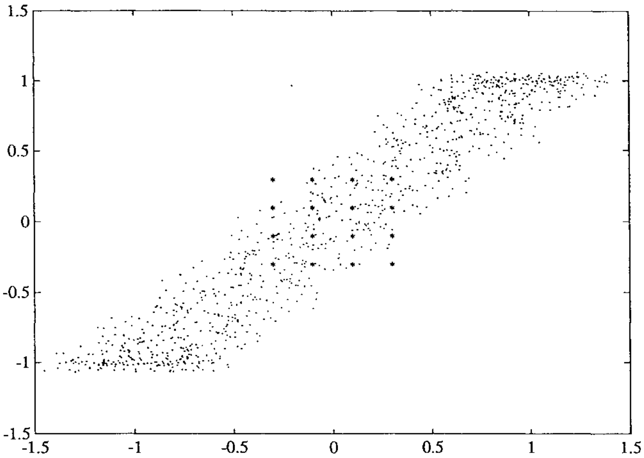

Fig. 7 Trajectory history and constant center grid (case 3).

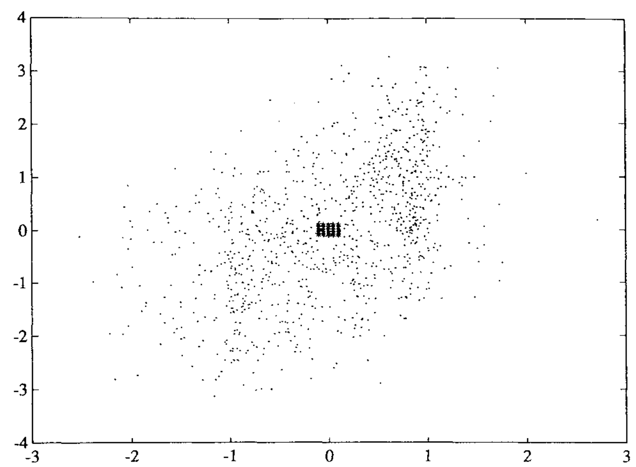

Fig. 8 Trajectory history and constant center grid (case 4).

the rate of convergence that can be achieved with adaptation of the grid of centers, as well as the feedback gains. The nonlinear system is governed by the following equations:

$$H_0 = (1/2m_2)p_2^2 + (1/2m_1)p_1^2 + \tfrac{1}{2}K_0(q_2 - q_1)^2 + \tfrac{1}{4}K_1(q_2 - q_1)^4 \tag{41a}$$

$$\dot{q}_1 = (1/m_1)p_1 \tag{41b}$$

$$\dot{q}_2 = (1/m_2)p_2 \tag{41c}$$

$$\dot{p}_1 = K_0(q_2 - q_1) + K_1(q_2 - q_1)^3 + u \tag{41d}$$

$$\dot{p}_2 = -K_0(q_2 - q_1) - K_1(q_2 - q_1)^3 \tag{41e}$$

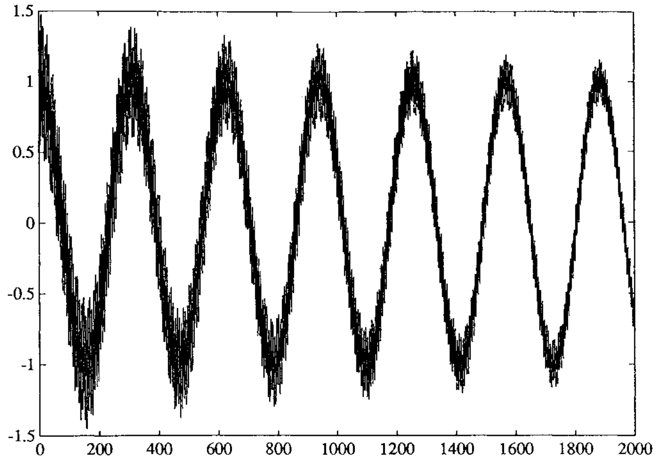

Fig. 9 Tracking trajectory with constant center grid (case 1).

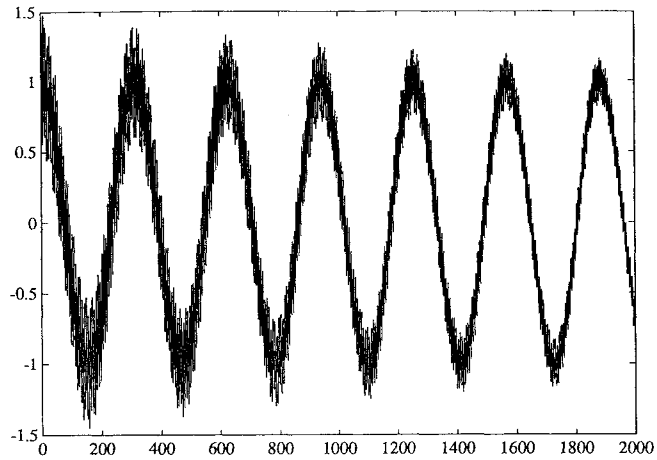

Fig. 10 Tracking trajectory with constant center grid (case 2).

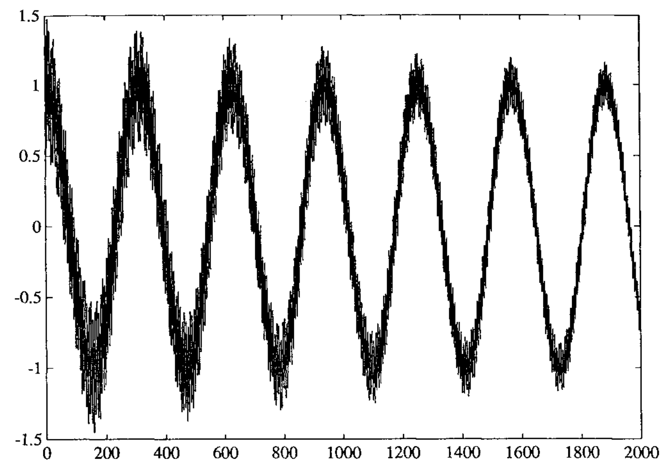

Fig. 11 Tracking trajectory with constant center grid (case 3).

This four-degree-of-freedom system will first be simulated with a number of grids of fixed centers in configuration space. The same system will then be subjected to the adaptation of centers of approximation algorithm. The target trajectory in each case is a simple sinusoid.

Figures 5–8 each show the discrete integrated trajectory

$$[q_1(t_k), q_2(t_k)]_{k=1}^{\infty}$$

plotted against the background of the grid of approximation centers (depicted as asterisks) in configuration space. The trajectories generally fall well outside of each grid of approximation centers.

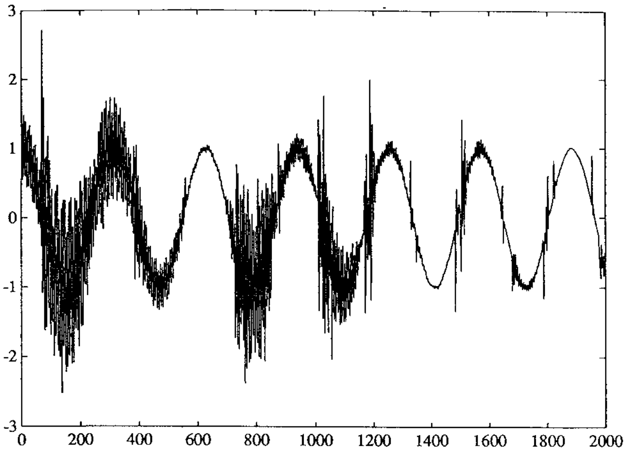

Fig. 12 Tracking trajectory with constant center grid (case 4).

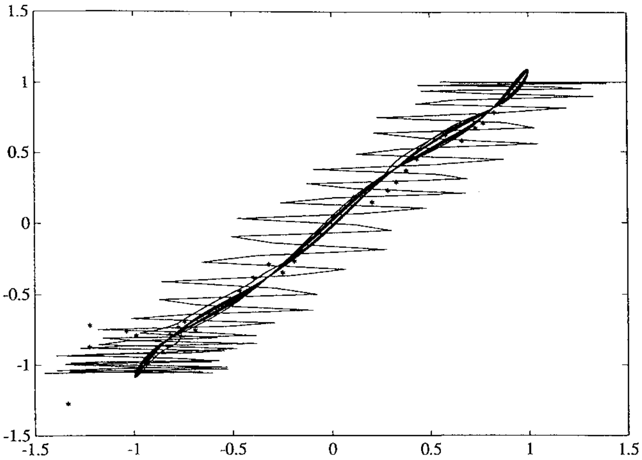

Fig. 13 Trajectory history with adaptive approximation centers.

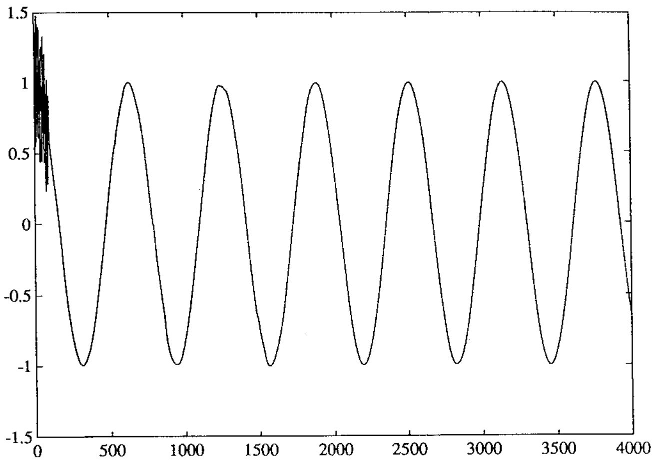

Fig. 14 Tracking trajectory for adaptive approximation centers.

Figures 9–12, corresponding to Figs. 5–8, respectively, are plots of the sinusoidal trajectory to be tracked. These figures show that tracking does in fact occur, although quite slowly.

Figure 13 is a plot of the integrated trajectory of the same system at an intermediate stage with the adaptation of centers algorithm employed. This figure also shows the centers of approximation at an intermediate point of the simulation to illustrate their motion. It is seen that the centers of approximation are moving toward the steady-state dynamics of the system, in this case a line in configuration space. The initial grid of approximation centers lay between the bounds of ± 1.0 in configuration space. The much more expedient convergence of the simulation to the desired trajectory is

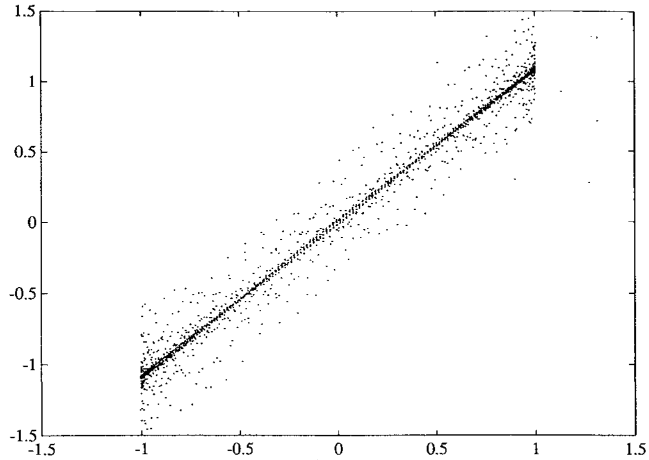

Fig. 15 Trajectory history for adaptive case showing system steady state.

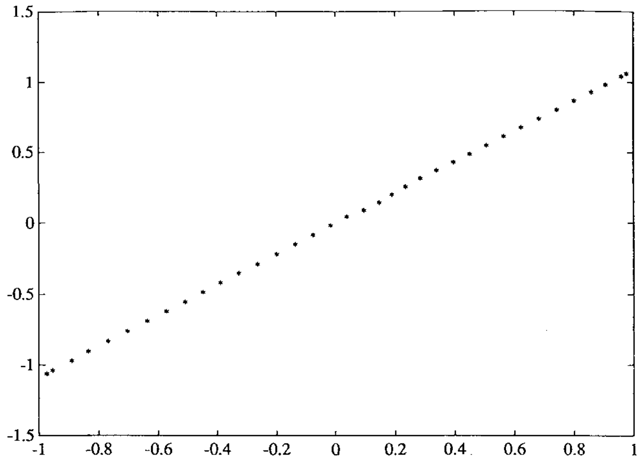

Fig. 16 Final position of centers of approximation for adaptive case.

shown by Fig. 14. A discrete mapping of the system trajectory over a longer period of time showing the system steady-state dynamics more pronouncedly follows in Fig. 15. The corresponding final grid of approximation centers is shown in Fig. 16.

Conclusions

An adaptive control strategy for a class of Hamiltonian control systems making use of radial basis function approximants has been presented. This originally fixed center of approximation strategy has been extended to not only adapt the feedback gains, but also the locations of the centers of approximation. This was accomplished using a learning vector quantization algorithm. The performance of nonlinear radial basis function control has been shown to be remarkably increased by the use of this adaptation of centers of approximation algorithm. The use of this algorithm is especially important when the system trajectories are not known a priori, because convergence of the tracking problem is delayed. This often results in a larger number of approximation centers being used, therefore greatly increasing the dimensionality of the problem. Finally, it is further discussed that the centers of approximation move so as to minimize the average quantization error of the system.

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